

JEE-Main-25-07-2022-Shift-1 (Memory Based)

MATHEMATICS

Question: $x^4 + x^3 + x^2 + x + 1 = 0$ equation has 4 roots $\alpha\beta\gamma\delta$. Then

$$\alpha^{2021} + \beta^{2021} + \gamma^{2021} + \delta^{2021} = ?$$

Options:

- (a) 4
- (b) 1
- (c) -1
- (d) 4

Answer: (c)

Solution:

$$\text{Given, } x^4 + x^3 + x^2 + x + 1 = 0$$

$$\Rightarrow x^5 + x^4 + x^3 + x^2 + x = 0$$

$$\Rightarrow x^5 - 1 = 0$$

$$\Rightarrow x^5 = 1$$

$$\alpha^{2021} + \beta^{2021} + \gamma^{2021} + \delta^{2021}$$

$$= \alpha(\alpha^5)^{400} + \beta(\beta^5)^{400} + \gamma(\gamma^5)^{400} + \delta(\delta^5)^{400}$$

$$= \alpha + \beta + \gamma + \delta$$

$$= -1$$

Question: A die is rolled twice and let the outcomes be α, β . Then probability such that

$$x^2 + \alpha x + \beta > 0$$

Options:

(a) $\frac{17}{36}$

(b) $\frac{9}{36}$

(c)

(d)

Answer: (a)

Solution:

$$x^2 + \alpha x + \beta > 0$$

$$\Rightarrow \alpha^2 - 4\beta < 0$$

$$\Rightarrow \alpha = 1, \beta = 1, 2, 3, 4, 5, 6$$

$$\Rightarrow \alpha = 2, \beta = 2, 3, 4, 5, 6$$

$$\Rightarrow \alpha = 3, \beta = 3, 4, 5, 6$$

$$\Rightarrow \alpha = 4, \beta = 5, 6$$

$$\Rightarrow \alpha = 5, \text{ not possible}$$

Favourable outcomes = $6 + 5 + 4 + 2 = 17$

$$\therefore p = \frac{17}{36}$$

Question: $\lim_{n \rightarrow \infty} \sqrt{n^2 - n + 1} + n\alpha + \beta = 0$ then $8(\alpha + \beta)$

Options:

(a) -8

(b) -4

(c)

(d)

Answer: (b)

Solution:

$$\lim_{n \rightarrow \infty} \sqrt{n^2 - n + 1} + n\alpha + \beta = 0$$

$$\lim_{n \rightarrow \infty} \left(\frac{\sqrt{n^2 - n + 1} + n\alpha + \beta}{\sqrt{n^2 - n + 1} - (n\alpha + \beta)} \right) \left(\sqrt{n^2 - n + 1} - (n\alpha + \beta) \right) = 0$$

$$\Rightarrow \lim_{n \rightarrow \infty} \frac{n^2 - n + 1 - (n\alpha + \beta)^2}{\sqrt{n^2 - n + 1} - (n\alpha + \beta)} = 0$$

$$\Rightarrow \lim_{n \rightarrow \infty} \frac{n^2(1 - \alpha^2) + n(-1 - 2\alpha\beta) + 1 - \beta^2}{\sqrt{n^2 - n + 1} - (n\alpha + \beta)} = 0$$

$$\Rightarrow 1 - \alpha^2 = 0$$

$$\Rightarrow \alpha = \pm 1$$

$$-1 - 2\alpha\beta = 0$$

$$\beta = \frac{-1}{2\alpha}$$

$$\alpha = 1, \beta = \frac{-1}{2}$$

$$\alpha = -1, \beta = \frac{1}{2}$$

$$8(\alpha + \beta) = \pm 4$$

Question: The number of solution of $|\cos x| = \sin x$ such that $-4\pi \leq x \leq 4\pi$ is

Options:

(a) 4

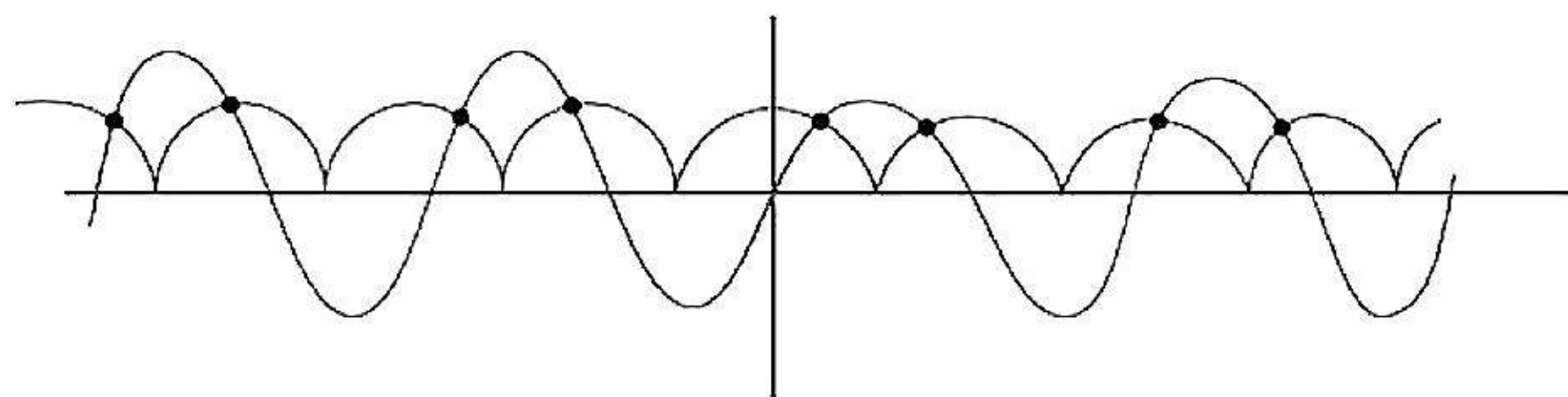
(b) 6

(c) 8

(d) 12

Answer: (c)

Solution:



Number of solutions = 8

Question: Which of the following is a tautology?

Options:

(a) $(\sim p \vee q) \Rightarrow p$

(b) $p \Rightarrow (\sim p \vee q)$

(c) $(\sim p \vee q) \Rightarrow q$

(d) $q \Rightarrow (\sim p \vee q)$

Answer: (d)

Solution:

$$q \Rightarrow (\sim p \vee q)$$

$$\sim q \vee (\sim p \vee q)$$

$$(\sim q \vee \sim p) \vee (\sim q \vee q)$$

$$(\sim q \vee \sim p) \vee (T)$$

$$T$$

Question: Set $A = \{1, 2, 3, 4\}$ and $B = \{1, 2, 3, 4, 5, 6\}$. Find the number of functions from A to B such that $f(1) + f(2) = f(3)$

Answer: 40.00

Solution:

$$A = \{1, 2, 3, 4\}, B = \{1, 2, 3, 4, 5\}$$

$$f(1) + f(2) = f(3)$$

$$1 + 2 = 3 \rightarrow 2 \times 5 = 10$$

$$1 + 3 = 4 \rightarrow 2 \times 5 = 10$$

$$1 + 4 = 5 \rightarrow 2 \times 5 = 10$$

$$2 + 3 = 5 \rightarrow 2 \times 5 = 10$$

$$\text{Number of functions} = 10 + 10 + 10 + 10 = 40$$

Question: If roots of $x^2 - 8ax + 2a = 0$ are p & r , while q & s are roots of

$x^2 + 12bx + 6b = 0$ then find $\frac{1}{a} - \frac{1}{b}$ if $\frac{1}{p}, \frac{1}{q}, \frac{1}{r}$ & $\frac{1}{s}$ are in AP.

Answer:

Solution:

$$p + r = 8a, pr = 2a$$

$$q + s = -12b, qs = 6b$$

$$\begin{aligned}
&\Rightarrow \frac{1}{p} + \frac{1}{r} = 4, \frac{1}{q} + \frac{1}{s} = -2 \\
&\Rightarrow \frac{1}{p} - \frac{1}{q} + \frac{1}{r} - \frac{1}{s} = 6 \\
&\Rightarrow -2d = 6 \\
&\Rightarrow d = -3 \\
&\frac{1}{p} + \frac{1}{p} - 3 = 4 \\
&\Rightarrow \frac{2}{p} = 7 \\
&\Rightarrow \frac{1}{p} = \frac{2}{7}, \frac{1}{q} = \frac{-19}{7}, \frac{1}{r} = \frac{-40}{7}, \frac{1}{s} = \frac{-61}{7} \\
&\frac{1}{a} = \frac{2}{pr} = 2 \cdot \frac{2}{7} \cdot \left(\frac{-40}{7} \right) = \frac{-80}{49} \\
&\frac{1}{b} = \frac{6}{qs} = 6 \cdot \left(\frac{-19}{7} \right) \left(\frac{-61}{7} \right) = \frac{6954}{49} \\
&\frac{1}{a} - \frac{1}{b} = \frac{-80}{49} - \frac{6954}{49} \\
&= \frac{-7034}{49}
\end{aligned}$$

Question: If $a_1 = b_1 = 1, a_n = a_{n-1} + 2$ & $b_n = a_n + b_{n-1}$, then find $\sum_{n=1}^{15} a_n \times b_n$

Answer: 27560.00

Solution:

$$a_1 = b_1 = 1, a_n = a_{n-1} + 2$$

$$a_r = 1 + (r-1)2 = 2r-1$$

$$b_r = a_r + b_{r-1} = a_r + a_{r-1} + b_{r-2}$$

$$= a_r + a_{r-1} + a_{r-2} + \dots + a_2 + b_1$$

$$= \sum (2r-1)$$

$$= \frac{2(r)(r+1)}{2} - r = r^2$$

$$\sum a_n b_n = \sum (2n-1)n^2$$

$$= \sum 2n^3 - \sum n^2$$

$$= 2 \left(\frac{15 \times 16}{2} \right)^2 - \frac{15 \times 16 \times 31}{6}$$

$$= 27560$$

Question: A line with slope greater than 1, passes through $A(4, 3)$. Line $x - y = 2$ intersects former line at B . Find B if $AB = \frac{\sqrt{29}}{3}$.

Answer: $\frac{17}{3}, \frac{11}{3}$

Solution:

$$\frac{x-4}{\cos \theta} = \frac{y-3}{\sin \theta} = r$$

$$x = \frac{\sqrt{29}}{3} \cos \theta + 4, y = \frac{\sqrt{29}}{3} \sin \theta + 3$$

$$x - y = 2$$

$$\Rightarrow \frac{\sqrt{29}}{3} \cos \theta + 4 - \frac{\sqrt{29}}{3} \sin \theta - 3 = 2$$

$$\Rightarrow \frac{\sqrt{29}}{3} (\cos \theta - \sin \theta) = 1$$

$$\Rightarrow \cos \theta - \sin \theta = \frac{3}{\sqrt{29}}$$

$$\Rightarrow 1 - \sin 2\theta = \frac{9}{29}$$

$$\Rightarrow \sin 2\theta = \frac{20}{29}$$

$$\Rightarrow \sin \theta + \cos \theta = \sqrt{1 + \left(\frac{20}{29}\right)}$$

$$\Rightarrow \sin \theta + \cos \theta = \frac{7}{\sqrt{29}}$$

$$\sin \theta = \frac{2}{\sqrt{29}}, \cos \theta = \frac{5}{\sqrt{29}}$$

$$x = \frac{\sqrt{29}}{3} \times \left(\frac{5}{\sqrt{29}}\right) + 4 = \frac{5}{3} + 4 = \frac{17}{3}$$

$$y = \frac{\sqrt{29}}{3} \times \frac{2}{\sqrt{29}} + 3 = \frac{2}{3} + 3 = \frac{11}{3}$$

$$B \equiv \left(\frac{17}{3}, \frac{11}{3}\right)$$

Question: Find remainder when $(2024)^{2024}$ is divided by 7.

Answer: 1.00

Solution:

$$(2024)^{2024}$$

$$= (289 \times + 1)^{2024}$$

$$= {}^{2024}C_0 + {}^{2024}C_1 (289 \times 7) + \dots$$

Remainder = 1

Question: $z = 1 + \cos \frac{6\pi}{5} + i \sin \frac{6\pi}{5}$, find arg.

Answer: $\frac{3\pi}{5}$

Solution:

$$\begin{aligned} z &= 1 + \cos \frac{6\pi}{5} + i \sin \frac{6\pi}{5} \\ &= 2 \cos^2 \frac{6\pi}{10} + 2i \sin \frac{6\pi}{10} \cos \frac{6\pi}{10} \\ &= 2 \cos \frac{3\pi}{5} \left(\cos \frac{3\pi}{5} + i \sin \frac{3\pi}{5} \right) \\ \arg(z) &= \frac{3\pi}{5} \end{aligned}$$

Question: Sum & product of mean & variance of Binomial distribution are 24 & 128 respectively. Find probability of 1 or 2 successes.

Answer:

Solution:

Mean and variance are roots of $x^2 - 24x + 128 = 0$

$$\Rightarrow \text{Mean} = np = 16$$

$$\text{Variance} = npq = 8$$

$$\Rightarrow p = q = \frac{1}{2}, n = 32$$

$$\begin{aligned} p(x=1) + p(x=2) &= {}^{32}C_1 \left(\frac{1}{2} \right)^{32} + {}^{32}C_2 \left(\frac{1}{2} \right)^{32} \\ &= \left({}^{32}C_1 + {}^{32}C_2 \right) \left(\frac{1}{2} \right)^{32} \\ &= {}^{33}C_2 \left(\frac{1}{2} \right)^{32} \end{aligned}$$

Question: Find dictionary rank of MANKIND.

Answer: 1492.00

Solution:

$$\begin{aligned} &\text{M} \quad \text{A} \quad \text{N} \quad \text{K} \quad \text{I} \quad \text{N} \quad \text{D} \\ &\frac{4 \cdot 6!}{2!} + 0 + \frac{3(4!)}{2!} + 2(3!) + 2! + 1 + 1 \\ &= 1440 + 36 + 12 + 2 + 1 + 1 \\ &= 1492 \end{aligned}$$

Question: If the locus of centre (α, β) of circle that touches both $x^2 + (y-1)^2 = 1$ & x-axis, then find area enclosed by locus & $y = 4$

Answer: $\frac{64}{3}$

Solution:

$$(x - \alpha)^2 + (y - \beta)^2 = \beta^2$$

$$|\beta + 1| = \sqrt{(\alpha)^2 + (\beta - 1)^2}$$

$$\Rightarrow \alpha^2 = 4\beta$$

$$\Rightarrow x^2 = 4y$$

$$\Rightarrow y = \frac{x^2}{4}$$

$$\text{Area} = 2 \int_0^4 \left(4 - \frac{x^2}{4} \right) = \frac{64}{3}$$

Question: $p(x) = x^3 + ax^2 + bx + c$, $y = p(x)$ touches x-axis at $(-2, 0)$ & $p'(0) = 3$. Find local maxima.

Answer: 0.00

Solution:

$$p(x) = x^3 + ax^2 + bx + c$$

$$p(-2) = 0$$

$$0 = -8 + 4a - 2b + c$$

$$p'(-2) = 0$$

$$\Rightarrow 12 - 4a + b = 0$$

$$p'(0) = 3$$

$$\Rightarrow c = 3$$

$$\Rightarrow b = 7$$

$$\Rightarrow a = \frac{19}{4}$$

$$p(x) = x^3 + \frac{19}{4}x^2 + 7x + 3$$

$$p'(x) = 3x^2 + \frac{19}{2}x + 7$$

$$x = \frac{\frac{-19}{2} \pm \sqrt{\left(\frac{19}{2}\right)^2 - 4 \cdot 7 \cdot 3}}{6}$$

$$= \frac{\frac{-19}{2} \pm \frac{5}{2}}{6} = -2, \frac{-7}{6}$$

$$p''(x) = 6x + \frac{19}{2}$$

$$p''(-2) = -12 + \frac{19}{2} < 0$$

$$p''\left(\frac{-7}{6}\right) > 0$$

$$\text{Maxima} = p(-2) = 0$$