

JEE-Main-26-07-2022-Shift-1 (Memory Based)

MATHEMATICS

Question: How many 5 digit number can be formed such that product of digits is 30.

Answer: 80.00

Solution:

$$x_1 \cdot x_2 \cdot x_3 \cdot x_4 \cdot x_5 = 30 = 5 \times 2 \times 3$$

$$1 \ 1 \ 2 \ 3 \ 5 \rightarrow \frac{5!}{2!} = 60$$

$$1 \ 1 \ 1 \ 6 \ 5 \rightarrow \frac{5!}{3!} = 20$$

$$60 + 20 = 80$$

Question: $f(3x) - f(x) = x$, $f(8) = 7$, find $f(14)$.

Answer: 10.00

Solution:

$$f(3x) - f(x) = x$$

$$f(x) - f\left(\frac{x}{3}\right) = \frac{x}{3}$$

$$f\left(\frac{x}{3}\right) - f\left(\frac{x}{3^2}\right) = \frac{x}{3^2}$$

$\vdots$

On adding, we get

$$f(x) - \lim_{n \rightarrow \infty} f\left(\frac{x}{3^n}\right) = x\left(\frac{1}{3} + \frac{1}{3^2} + \dots + \infty\right)$$

$$\Rightarrow f(x) - f(0) = \frac{x}{2}$$

$$\because f(8) = 7, \text{ so } f(0) = 3$$

$$\therefore f(x) = \frac{x}{2} + 3$$

$$\therefore f(14) = 10$$

Question: Coefficient of x & x^2 in $(1+x)^p \times (1-x)^q$ are -3 & -5 respectively. Find coefficient of x^3 .

Answer: 23.00

Solution:

Given $(1+x)^p \times (1-x)^q$

$$\left({}^p C_0 + {}^p C_1 x + {}^p C_2 x^2 + {}^p C_3 x^3 \dots\right) \left({}^q C_0 - {}^q C_1 x + {}^q C_2 x^2 - {}^q C_3 x^3 \dots\right)$$

$$p - q = -3$$

$$-pq + \frac{q(q-1)}{2} + \frac{p(p-1)}{2} = -5$$

$$-2pq + q^2 - q + p^2 - p = -10$$

$$(p-q)^2 - p - q = -10$$

$$9 - p - q = -10$$

$$p + q = 19$$

$$\Rightarrow p = 8, q = 11$$

Coefficient of $x^3 = -{}^q C_3 + {}^p C_3 + p {}^q C_2 - q {}^p C_2$

$$= -{}^{11} C_3 + {}^8 C_3 + 8 {}^{11} C_2 - 11 {}^8 C_2$$

$$= \frac{-11 \cdot 10 \cdot 9}{6} + \frac{8 \cdot 7 \cdot 6}{6} + \frac{8 \cdot 11 \cdot 10}{2} - \frac{11 \cdot 8 \cdot 7}{2}$$

$$= 23$$

Question: $\frac{dy}{dx} + (2 \tan x) y = \sin x, y\left(\frac{\pi}{3}\right) = 0$, find $f(x)|_{\max}$

Answer: $\frac{1}{8}$

Solution:

$$\frac{dy}{dx} + (2 \tan x) y = \sin x$$

$$\text{IF} = e^{\int 2 \tan x dx} = e^{-2 \ln \cos x} = \frac{1}{\cos^2 x}$$

$$\frac{y}{\cos^2 x} = \int \frac{\sin x}{\cos^2 x} dx$$

$$\frac{y}{\cos^2 x} = \frac{1}{\cos x} + C$$

$$0 = \frac{1}{\cos \frac{\pi}{3}} + C$$

$$C = -2$$

$$y = \cos x - 2 \cos^2 x$$

$$= -2 \left[\cos^2 x - \frac{1}{2} \cos x \right]$$

$$= -2 \left[\cos^2 x - \frac{1}{2} \cos x + \frac{1}{16} - \frac{1}{16} \right]$$

$$= -2 \left[\cos x - \frac{1}{4} \right]^2 + \frac{1}{8}$$

$$y_{\max} = \frac{1}{8}$$

Question: Find sum of elements in 11th term: (3);(6,9,12);(15,18,21,24,27);.....

Answer: 6993.00

Solution:

(3);(6,9,12);(15,18,21,24,27);.....

$$1+3+5+....10 \text{ terms} = \frac{10}{2} [2 \times 1 + (10-1)2]$$

$$= 100$$

$$a_{146} = 3 + (99)3 = 300$$

$$11^{\text{th}} \text{ term} = (303 +21 \text{ terms})$$

$$= \frac{21}{2} [2 \times 303 + (20)3]$$

$$= 6993$$

Question: $\tan \left[2 \tan^{-1} \left(\frac{1}{5} \right) + \sec^{-1} \left(\frac{\sqrt{5}}{2} \right) + 2 \tan^{-1} \left(\frac{1}{8} \right) \right] = ?$

Answer: 2.00

Solution:

$$2 \left(\tan^{-1} \left(\frac{1}{8} \right) + \tan^{-1} \left(\frac{1}{5} \right) \right) = 2 \tan^{-1} \left(\frac{\frac{1}{8} + \frac{1}{5}}{1 - \frac{1}{40}} \right)$$

$$= 2 \tan^{-1} \left(\frac{1}{3} \right)$$

$$= \tan^{-1} \left(\frac{\frac{2}{3}}{1 - \frac{1}{9}} \right)$$

$$= \tan^{-1} \left(\frac{3}{4} \right)$$

∴ The given terms reduces to

$$\tan \left(\tan^{-1} \left(\frac{3}{4} \right) + \tan^{-1} \left(\frac{1}{2} \right) \right)$$

$$= \tan \left(\tan^{-1} \frac{\frac{3}{4} + \frac{1}{2}}{1 - \frac{3}{8}} \right)$$

$$= 2$$

Question: $x + y - z = 0 = x - 2y + 3z - 5$. There is a line parallel to this & passing through $(1, -2, 3)$. Find distance of this line from $(1, 4, 5)$.

Answer: 0

Solution:

$$x + y - z = 0, x - 2y + 3z - 5 = 0$$

$$n = \begin{vmatrix} i & j & k \\ 1 & 1 & -1 \\ 1 & -2 & 3 \end{vmatrix} = i(3 - 2) - j(3 + 1) + k(-2 - 1)$$

$$= i - 4j - 3k$$

$$\text{Line} \Rightarrow \frac{x-1}{1} = \frac{y+2}{-4} = \frac{z-3}{-3} = \lambda$$

$$P \equiv x = \lambda + 1, y = -4\lambda + 2, z = -3\lambda + 3$$

$$PQ = (\lambda, -4\lambda - 2, -3\lambda - 2)$$

$$\text{Now, } \lambda - 4(-4\lambda - 2) - 3(-3\lambda - 2) = 0$$

$$\lambda + 16\lambda + 8 + 9\lambda + 6 = 0$$

$$26\lambda + 14 = 0$$

$$\lambda = \frac{-14}{26} = \frac{-7}{13}$$

$$PQ = \sqrt{\lambda^2 + (-4\lambda - 2)^2 + (-3\lambda - 2)^2}$$

$$= \sqrt{\lambda^2 + 16\lambda^2 + 16\lambda + 4 + 9\lambda^2 + 12\lambda + 4}$$

$$= \sqrt{26\lambda^2 + 18\lambda + 4}$$

$$= \sqrt{26\left(\frac{-7}{13}\right)^2 + 18\left(\frac{-7}{13}\right) + 4}$$

$$= \sqrt{\frac{2 \times 49}{13} - \frac{7 \times 18}{13} + 4}$$

$$= \sqrt{\frac{24}{13}}$$

Question: Area under $y = f(x)$ from 3 to x (where $x > 3$) is $\left(\frac{y}{x}\right)^3 \cdot f(3) = 3$ then for

$y = 6\sqrt{10}$, what will be x ?

Answer: 6.00

Solution:

$$\int_3^x f(t) dt = \left(\frac{y}{x}\right)^3$$

$$f(x) = 3\left(\frac{y}{x}\right)^2 \left(\frac{y'x - y}{x^2}\right)$$

$$y = \frac{3y^2}{x^2} \left(\frac{y'x - y}{x^2}\right)$$

$$\Rightarrow x^4 = 3y(y'x - y)$$

$$\Rightarrow x^2 dx = 3y \left(\frac{xdy - ydx}{x^2}\right)$$

$$\Rightarrow x^2 dx = 3y d\left(\frac{y}{x}\right)$$

$$\Rightarrow xdx = 3\frac{y}{x} d\left(\frac{y}{x}\right)$$

$$\Rightarrow \frac{x^2}{2} = \frac{3}{2} \left(\frac{y}{x}\right)^2 + C$$

$$\Rightarrow x^2 = \frac{3y^2}{x^2} + C$$

$$\Rightarrow 3^2 = 3 + C$$

$$\Rightarrow C = 6$$

$$\Rightarrow x^2 = \frac{3y^2}{x^2} + 6$$

$$y = 6\sqrt{10}$$

$$\Rightarrow x^2 = \frac{3 \cdot 36 \times 10}{x^2} + 6$$

$$\Rightarrow x = 6$$

Question: From a group of 10 boys $B_1, B_2, \dots, B_{10}$ and 5 girls $G_1, G_2, \dots, G_5$, the number of ways of selection of group of 3 boys and 3 girls, such that B_1 & B_2 are not together in group is ____.

Answer: 1120.00

Solution:

Number of ways to select 3 boys = Total ways – No. of ways when both B_1 & B_2 are selected

$$= {}^{10}C_3 - {}^8C_1 = 112$$

Number of ways to select 3 girls = ${}^5C_3 = 10$

Required number of ways = $112 \times 10 = 1120$

Question: $f(x) = \sqrt{\frac{1 - \cos 2x}{1 + \cos 2x}}$, find relation between $f\left(\frac{a}{2}\right)$ & $f'\left(\frac{a}{2}\right)$

Options:

(a) $\sqrt{2f\left(\frac{a}{2}\right)} = f'\left(\frac{a}{2}\right)$

(b)

(c)

(d)

Answer: ()

Solution:

$$f(x) = \sqrt{\frac{1 - \cos 2x}{1 + \cos 2x}}$$

$$f(x) = \sqrt{\frac{\sin^2 x}{\cos^2 x}} = \tan x$$

$$f'(x) = \sec^2 x = 1 + \tan^2 x$$

$$f'(x) = 1 + f^2(x)$$

$$f'\left(\frac{a}{2}\right) = 1 + f^2\left(\frac{a}{2}\right)$$

Question: $f(x) = \begin{cases} \frac{\ln(1-x+x^2) + \ln(1+x+x^2)}{\sec x - \cos x} & ; x < 0 \\ k & ; x \geq 0 \end{cases}$ is continuous, find k .

Answer: 1.00

Solution:

$$\lim_{x \rightarrow 0} \frac{\ln(1-x+x^2) + \ln(1+x+x^2)}{\sec x - \cos x}$$

$$\lim_{x \rightarrow 0} \frac{\frac{1}{1-x+x^2}(-1+2x) + \frac{1}{1+x+x^2}(1+2x)}{\sec x \tan x + \sin x}$$

$$\lim_{x \rightarrow 0} \frac{(2x-1)(1+x+x^2) + (2x+1)(1-x+x^2)}{(1+x^2+x^4)(\sec x \tan x + \sin x)}$$

$$= \lim_{x \rightarrow 0} \frac{2x + 2x^2 + 2x^3 - 1 - x - x^2 + 2x - 2x^2 + 2x^3 + 1 - x + x^2}{(1+x^2+x^4)(\sec x \tan x + \sin x)}$$

$$= \lim_{x \rightarrow 0} \frac{4x^3 + 2x}{(1+x^2+x^4)(\sec^2 x \sin x + \sin x)}$$

$$= \lim_{x \rightarrow 0} \frac{4x^2 + 2}{(1+x^2+x^4)(\sec^2 x + 1)}$$

$$= \frac{2}{1 \times 2} = 1$$

Question: Normal to $y^2 = 24x$ at (α, β) is perpendicular to $2x + 2y = 5$. Find equation of normal to $\frac{x^2}{\alpha^2} - \frac{y^2}{\beta^2} = 1$ at $(\alpha + 4, \beta + 10)$.

Answer: ()

Solution:

$$\frac{x^2}{\alpha^2} - \frac{y^2}{\beta^2} = 1 \quad (\alpha + 4, \beta + 10)$$

$$\beta^2 = 24\alpha$$

$$2yy' = 24$$

$$y' = \frac{12}{y} = \frac{12}{\beta}$$

$$\Rightarrow \frac{12}{\beta} = 1 \Rightarrow \beta = 12$$

$$12^2 = 24 \cdot \alpha$$

$$\Rightarrow \alpha = 6$$

$$\frac{x^2}{36} - \frac{y^2}{144} = 1$$

$$\frac{2x}{36} - \frac{2yy'}{144} = 0$$

$$\frac{x}{36} = \frac{yy'}{144}$$

$$\Rightarrow y' = \frac{144}{136} \cdot \frac{x}{y} = \frac{4x}{y} = \frac{4 \times 10}{22}$$

$$= \frac{20}{11}$$

$$y - 22 = \frac{20}{11}(x - 10)$$

$$11y - 242 = 20x - 200$$

$$\Rightarrow 20x - 11y + 42 = 0$$