JEE-Main-27-07-2022-Shift-1 (Memory Based)

MATHEMATICS

Question: Let $a_1, a_2, a_3, ..., a_n$ be in A.P. The ratio of sum of first five term to the sum of first nine terms is $5:17$. Also $110 < a_{15} < 120$. Find the sum of first 10 terms of the A.P. (where all $a_i\ (i = 1, 2, 3, ..., n)$ are integers)

Options:
(a) 330
(b) 460
(c) 290
(d) 380

Answer: (d)

Solution:

Let first term be 'a' and common difference be 'd' for the A.P.

$$\frac{S_5}{S_9} = \frac{5}{17}$$

$$\Rightarrow \frac{\frac{5}{2}(2a+4d)}{\frac{9}{2}(2a+8d)} = \frac{5}{17}$$

$$\therefore 4a = d$$

$$a_{15} = a + 14d = 57a$$

It is given that $110 < a_{15} < 120$

$$\Rightarrow 110 < 57a < 120$$

For integral terms of the A.P, $a = 2$

Sum of 10 terms of A.P:

$$S_{10} = \frac{10}{2}(4 + 9 \times 8) = 380$$

Question: Let S be sample space for 5 digit numbers. If p is probability of a number being randomly selected which is multiple of 7 but not divisible by 5, then $9p$ is equal to:

Options:
(a) 1.0146
(b) 1.2085
(c) 1.0285
(d) 1.1521

Answer: (c)

Solution:

Five digit number line from 10000 to 99999

$$\therefore S = 90000$$

Number divisible by 7 $=\dfrac{90000}{7}$

Number divisible by 7 and multiple by 5 $=\dfrac{90000}{35}$

$\therefore$ Required Probability $=\dfrac{\dfrac{90000}{7}-\dfrac{90000}{35}}{90000}$

$\Rightarrow p=\dfrac{4}{35}$

$\therefore 9p=\dfrac{36}{35}=1.02857$

Question: Let $A=\begin{bmatrix}1 & 2\\-2 & -5\end{bmatrix}$, α & β belongs to real numbers such that $\alpha A^2+\beta A=2I$, where I is an identity matrix of order 2×2. Then the value of $\alpha+\beta$ is equal to:

Options:

(a) -10

(b) -6

(c) 6

(d) 10

Answer: (d)

Solution:

$A=\begin{bmatrix}1 & 2\\-2 & -5\end{bmatrix}$

$A^2=\begin{bmatrix}1 & 2\\-2 & -5\end{bmatrix}\begin{bmatrix}1 & 2\\-2 & -5\end{bmatrix}=\begin{bmatrix}-3 & -8\\8 & 21\end{bmatrix}$

It is given that $\alpha A^2+\beta A=2I$

$\alpha\begin{bmatrix}-3 & -8\\8 & 21\end{bmatrix}+\beta\begin{bmatrix}1 & 2\\-2 & -5\end{bmatrix}=2\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$

$-3\alpha+\beta=2$ & $-8\alpha+2\beta=0$

$\alpha=2,\ \beta=8$

$\Rightarrow \alpha+\beta=10$

Question: $(p\wedge r)\Leftrightarrow(p\wedge\sim q)$ which is equivalent to $\sim p$. Then r will be:

Options:

(a) p

(b) $\sim p$

(c) q

(d) $\sim q$

Answer: (c)

Solution:

The truth table is

p	q	$\sim p$	$\sim q$	$p \wedge q$	$p \wedge \sim q$	$p \wedge q \Leftrightarrow p \wedge \sim q$
T	T	F	F	T	F	F
T	F	F	T	F	T	F
F	T	T	F	F	F	T
F	F	T	T	F	F	T

Clearly $(p \wedge r) \Leftrightarrow (p \wedge \sim q) \equiv \sim p$

$\therefore r = q$

Question: The remainder of $(2021)^{2022} + (2022)^{2021}$ when divided by 7 is:

Answer: 0.00

Solution:

Let $S = (2021)^{2022} + (2022)^{2021}$

$\Rightarrow (2023-2)^{2022} + (2023-1)^{2021}$

$= 7k_1 + 2^{2022} + 7k_2 - 1$

$= 7(k_1 + k_2) + 8^{674} - 1$

$= 7(k_1 + k_2) + (7+1)^{674} - 1$

$= 7(k_1 + k_2) + 7k_3 + 1 - 1$

$= 7(k_1 + k_2 + k_3)$

Therefore, S is divisible by 7

Question: The mean and variance of 10 observation was 15 and 15. The mistake was 25 instead of 15. The new standard deviation is:

Answer: ()

Solution:

$$\frac{x_1 + ... + x_9 + 25}{10} = 15$$

$$\frac{x_1 + ... + x_9 + 15}{10} = \text{ correct mean } = m$$

$$\frac{25}{10} - \frac{15}{10} = 15 - m$$

$m = 4$

Correct mean is 4.

$$\frac{x_1^2 + ... + x_9^2 + 25^2}{10} - 15^2 = 15$$

$$\frac{x_1^2+....+x_9^2+15^2}{10}-14^2 = \text{correct variance} = \text{new SD}$$

$$\frac{25^2-15^2}{10}-\left(15^2-14^2\right)=15-v$$

Variance is 4, SD is 2.

Question: Let $f(x)=2x^2-x-1$ and $S=\{n:|f(n)|\le 800\}$ where $n\in z$, then $\sum_{n\in s} f(n)=$

Answer: 10620.00

Solution:

$-800\le f(n)\le 800$

$-800\le 2n^2-n-1\le 800$

$2n^2-n+799\ge 0$

$a>0$

$D=1-4(2)(799)<0$

Always true

$n\in R$

$2n^2-n-801\le 0$

$$n=\frac{1\pm\sqrt{1+4(2)(801)}}{4}$$

$$=\frac{1\pm\sqrt{6408}}{4}$$

$$n\approx\frac{1\pm 80}{4}$$

$$n=\frac{-79}{4},\frac{81}{4}$$

$n\in[-19.75,\ 20.25]$

$n\in\{-19,-18,-17,.....,-1,0,1,.....,20\}$

$f(n)=2n^2-n-1$

$f(-19)=2(-19)^2-(-19)-1$

$\vdots$

$f(19)=2(19)^2-(19)-1$

$f(20)=2(20)^2-(20)-1$

$=2\left[(-19)^2+(-18)^2+...(-1)^2+0^2+(1)^2+...+19^2+20^2\right]$

$=-\left[(-19)+(-18)+...+(-1)+0+(1)+...+(19)+(20)\right]-40$

$$= 2\left[400 + 2\left(\frac{19 \times 20 \times 39}{6}\right)\right] - 20 - 40$$

$$= 10620$$