

JEE-Main-28-07-2022-Shift-2 (Memory Based)

Physics

Question: Two photons, first has energy twice the other. If energy of first photon is 5 times its work function, find the ratio of maximum velocities

Options:

(a) $\frac{3}{2}$

(b) $\frac{2}{3}$

(c) $\frac{4}{3}$

(d) $\frac{4}{9}$

Answer: (b)

Solution:

Max. K.E. of electron

$$= E - \phi$$

$$\frac{1}{2}mv_1^2 = E_1 - \phi$$

$$\frac{1}{2}mv_2^2 = E_2 - \phi$$

$$\frac{v_1^2}{v_2^2} = \frac{5\phi - \phi}{10\phi - \phi} = \frac{4}{9}$$

$$\frac{v_1}{v_2} = \sqrt{\frac{4}{9}} = \frac{2}{3}$$

Question: A capacitor has capacity C_0 . When a dielectric of constant K & width $3/4^{\text{th}}$ of separation between plates is inserted, then find the new capacity.

Options:

(a) $\left(\frac{7k}{K+3}\right)C_0$

(b) $\left(\frac{5k}{K+3}\right)C_0$

(c) $\left(\frac{5k}{K+3}\right)C_0$

(d) $\left(\frac{4k}{K+3}\right)C_0$

Answer: (d)

Solution:

$$C = \frac{\epsilon_0 A}{\left(d - t + \frac{t}{k}\right)} \text{ and } C_0 = \frac{\epsilon_0 A}{d}$$

$$\text{Here } t = \frac{3}{4}d$$

$$\begin{aligned} \therefore C &= \frac{\epsilon_0 A}{\left(d - \frac{3}{4}d + \frac{3d}{4k}\right)} = \frac{\epsilon_0 A}{\frac{d}{4k}(3+k)} \\ &= \frac{4k}{(k+3)} C_0 \end{aligned}$$

Question: A chain of uniform mass density with length 'L', mass M is hanging over an ideal smooth pulley. When $l = \frac{L}{x}$, the acceleration of chain is $\frac{g}{2}$, find x?

Options:

(a) $\frac{L}{4}$

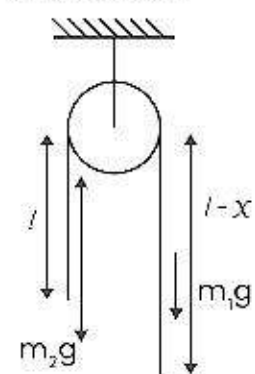
(b) $\frac{L}{2}$

(c) $\frac{L}{3}$

(d) $\frac{L}{1}$

Answer: (a)

Solution:



$$m_1 = \frac{M}{L}(L-x)g$$

$$m_2 = \frac{M}{L}xg$$

$$a = \frac{m_1g - m_2g}{M} = \frac{g}{2} \frac{\frac{M}{2}(L-x)g - \frac{M}{2}xg}{M} = \frac{g}{2}$$

$$\frac{L-x-x}{L} = \frac{1}{2}$$

$$L-2x = \frac{L}{2}$$

$$2x = L - \frac{L}{2} = \frac{L}{2} \Rightarrow x = \frac{L}{4}$$

Question: Time period of a pendulum is measured on surface of a planet with a stop watch of 1 sec resolution is found to be 200 sec. Its length was measured as $1 \pm 0.001m$. percentage error in measurement of arc due to gravity is

Options:

(a) 1.1%

(b) 0.3%

(c) 2.5%

(d) 5.2%

Answer: (a)

Solution:

$$\begin{aligned} \text{as } T &= 2\pi \sqrt{\frac{l}{g}} & \therefore \Delta g \% &= 2 \times \frac{1}{200} \times 100 + \frac{0.001}{1} \times 100 \\ \text{or } \frac{\Delta g}{g} &= \frac{2\Delta T}{T} + \frac{\Delta l}{l} & &= 1 + 0.1 \\ & & &= 1.1\% \end{aligned}$$

Question: Water flows out of a pipe and hits a wall, it has horizontal velocity v and cross-sectional area A . The density of the water is ρ . The water does not rebound from the wall. What is the force exerted on the wall by the water?

Options:

(a) $F = \rho v^2 A$

(b) $F = \rho v^1 A$

(c) $F = \rho v A$

(d) $F = \rho v^2$

Answer: (a)

Solution:

$$F = \frac{d}{dt} [\vec{p}_{\text{of water}}] = \left(\frac{dm}{dt} \right) v$$

$$= \rho V A v$$

$$\boxed{F = \rho v^2 A}$$

Question: Time period of a pendulum is measured on surface of a planet with a stop watch of 1 sec resolution is found to be 200 sec. Its length was measured as $1 \pm 0.001m$. Percentage error in measurement of arc due to gravity is

Options:

- (a) 1.4%
- (b) 1.1%
- (c) 2.2%
- (d) 2.1%

Answer: (b)

Solution:

$$T = 2\pi \sqrt{\frac{l}{g}}$$

$$\Rightarrow g = \frac{T^2}{4\pi^2 l}$$

$$\frac{\Delta g}{g} = 2 \frac{\Delta T}{T} + \frac{\Delta l}{l}$$

$$= 2 \left(\frac{1}{200} \right) + \frac{0.001}{1}$$

$$= \frac{1}{100} + 0.001 = 0.011$$

$$\% \text{ error} = 1.1\%$$

Question: Ball thrown upwards from a building with speed 19.6 m/s. Find maximum height achieved from point of projection? ($g = 9.8 \text{ m/s}^2$)

Options:

- (a) 1.96 m
- (b) 2.9 m
- (c) 19.6 m
- (d) 12.96 m

Answer: (c)

Solution:

Maximum height = h (say)

$$v^2 - u^2 = 2gh$$

At topmost point $v = 0$

And $u = 19.6 \text{ m/s}$ (given)

$$0^2 - (19.6)^2 = -2(9.8)(h)$$

$$h = \frac{(19.6)^2}{19.6} = 19.6 \text{ m}$$

Question: Time period of a simple pendulum on earth's surface is 4 sec and at height h above the earth's surface is 6 sec. Find the value of h if $R = 6400 \text{ km}$.

Options:

- (a) 1200
- (b) 2400

(c) 3200

(d) 3500

Answer: (c)

Solution:

$$T = 2\pi \sqrt{\frac{l}{g}}$$

$$\therefore \frac{T_1}{T_2} = \sqrt{\frac{g_2}{g_1}} = \sqrt{\frac{\frac{GM}{(R+h)^2}}{\frac{GM}{R^2}}}$$

$$\frac{4}{6} = \frac{R}{R+h}$$

$$\Rightarrow 4R + 4h = 6R \Rightarrow 4h = 2R$$

$$h = \frac{R}{2} = 3200$$

Question: For the given equation, please select incorrect statement, where K is Boltzmann constant, T is absolute temperature.

$$\sin \theta = \alpha \beta \ln \left[\frac{Bx}{KT} \right]$$

Options:

- (a) Dimension of β is same as force
- (b) Dimension of $\alpha^{-1}x$ is same as energy
- (c) Dimension of α and β is same
- (d) Dimension of KT is same as energy

Answer: (c)

Solution:

$$[B] = \frac{[KT]}{[x]} = \frac{M^1 L^2 T^{-2}}{L} = [M^1 L^1 T^{-2}]$$

$$\alpha = \frac{M^0 L^0 T^0}{\beta} = [M^{-1} L^{-1} T^{+2}]$$

Question: Assertion: Resistance of 80Ω is cut equally in 4 parts and all resistances are kept parallelly the net resistance is 5Ω

Reason: When $2R$ and $3R$ connected in parallel, ratio of heat dissipated in them is $3 : 2$
correct statement is/are

Options:

- (a) If both assertion and reason are true and the reason is the correct explanation of the assertion.
- (b) If both assertion and reason are true, but the reason is not the correct explanation of the assertion.
- (c) If assertion is true, but reason is false.
- (d) If both the assertion and reason are true.

Answer: (d)

Solution:

Assertion: Let r be resistance of each cut part then $r = \frac{80}{4} = 20\Omega$

Now 4 resistors of 20Ω in parallel $R_{eff} = \frac{20}{4} = 5\Omega$

$$\text{Reason: } \frac{H_1}{H_2} = \frac{\frac{V^2}{R_1}}{\frac{V^2}{R_2}} = \frac{R_2}{R_1} = \frac{3}{2}$$

Question: Two light rays incident on surface of a metal with energies 5 times and 6 times the work function ϕ of metal respectively. The ratio of speeds of electrons, ejected with maximum kinetic energies in each case, respectively is

Options:

(a) $\frac{4}{5}$

(b) $\frac{2}{5}$

(c) $\frac{4}{\sqrt{5}}$

(d) $\frac{2}{\sqrt{5}}$

Answer: (d)

Solution:

$$\text{As } KE_{\max} = \frac{1}{2}mv^2 = hf - W$$

$$\therefore \frac{1}{2}mv_1^2 = 5W - W = 4W$$

$$\text{And } \frac{1}{2}mv_2^2 = 6W - W = 5W$$

$$\therefore \left(\frac{V_1}{V_2}\right)^2 = \frac{4}{5}$$

$$\text{Or } \frac{V_1}{V_2} = \frac{2}{\sqrt{5}}$$

Question: Work done by gas in expansion is 150J and degrees of freedom = 8 find heat (q)

Options:

(a) 250

(b) 350

(c) 650

(d) 750

Answer: (d)

Solution:

$$W = p\Delta V = nR\Delta T = 150J \dots(1)$$

$$Q = nC_p\Delta T$$

$$= n\left(\frac{f+2}{2}\right)R\Delta T$$

$$= \left(\frac{8+2}{2}\right)150 = 5 \times 150 = 750$$

Question: Work done by gas in isobaric expansion is 150J and degrees of freedom = 8. Find heat (q)

Options:

(a) 750 J

(b) 650 J

(c) 550 J

(d) 450 J

Answer: (a)

Solution:

$$\Delta U = \frac{nf}{2}R\Delta T = 4nR\Delta T$$

$$\Delta W = 150J = nR\Delta T$$

$$\Delta Q = \Delta W + \Delta U$$

$$= nR\Delta T + 4nR\Delta T$$

$$= 5nR\Delta T$$

$$= 5(150) = 750J$$

Question: The ratio of magnetic field at the centre of circular coil and at a distance of $\sqrt{3}R$ from centre on axis of the coil is

Options:

(a) 1: 1

(b) 4 :2

(c) 7 : 3

(d) 8 : 1

Answer: (d)

Solution:

$$B_{\text{centre}} = \frac{\mu_0 i}{2R}$$

$$B_{\text{axis}} = \frac{\mu_0 i R^2}{2(R^2 + x^2)^{3/2}}$$

$$= \frac{\mu_0 i R^2}{2(3R^2 + R^2)^{3/2}} = \frac{\mu_0 i}{16R}$$

$$\Rightarrow \frac{B_{centre}}{B_{axis}} = \frac{8}{1}$$

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Chemistry

Question: Conc. HNO_3 reacts with I_2 to give

Options:

- (a) HI
- (b) HOI
- (c) HIO_3
- (d) HIO_2

Answer: (c)

Solution: Iodine reacts with concentrated nitric acid according to the following equation



Question: White phosphorus reacts with thionyl chloride to give

Options:

- (a) POCl_3
- (b) PCl_3
- (c) SO_2Cl_2
- (d) SCl_3

Answer: (c)



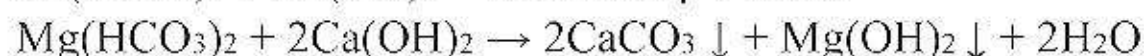
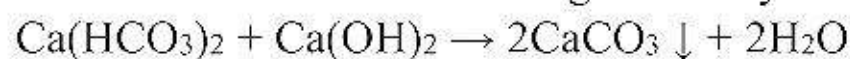
Question: Products obtained during treatment of hard water using Clark's method are

Options:

- (a) $\text{CaCO}_3, \text{Mg}(\text{OH})_2$
- (b) $\text{Ca}(\text{OH})_2, \text{MgCO}_3$
- (c) $\text{CaCO}_3, \text{MgCO}_3$
- (d) $\text{Ca}(\text{OH})_2, \text{Mg}(\text{OH})_2$

Answer: (a)

Solution: In Clark's method calculated amount of lime is added to hard water. It precipitates out calcium carbonate and magnesium hydroxide which can be filtered off.



Question: Decreasing order of metallic character

$\text{Na}, \text{Mg}, \text{Be}, \text{Si}, \text{P}$

Options:

- (a) $\text{Na} > \text{Mg} > \text{Be} > \text{P} > \text{Si}$
- (b) $\text{Mg} > \text{Na} > \text{Si} > \text{Be} > \text{P}$
- (c) $\text{Na} > \text{Mg} > \text{Be} > \text{Si} > \text{P}$
- (d) $\text{P} > \text{Si} > \text{Be} > \text{Mg} > \text{Na}$

Answer: (c)

Solution: Metallic character increases down the group and decreases along the period as we move left to right.

Hence, order of decreasing metallic character is $\text{Na} > \text{Mg} > \text{Be} > \text{Si} > \text{P}$

Question: Assertion: Permanganate titrations are not performed in presence of HCl

Reason: Chlorine is formed as a consequence of oxidation of HCl

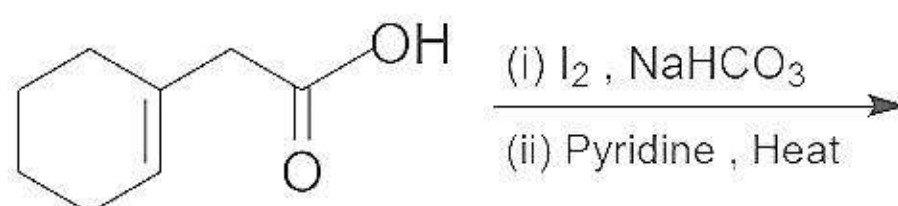
Options:

- (a) Both assertion and reason are true, reason is correct explanation of assertion.
- (b) Both assertion and reason are true, but reason is not a correct explanation of the assertion.
- (c) Assertion is true, but reason is false
- (d) Assertion is false, but reason is true

Answer: (a)

Solution: Permanganate titrations are not performed in presence of HCl because HCl oxidises to form chlorine.

Question:

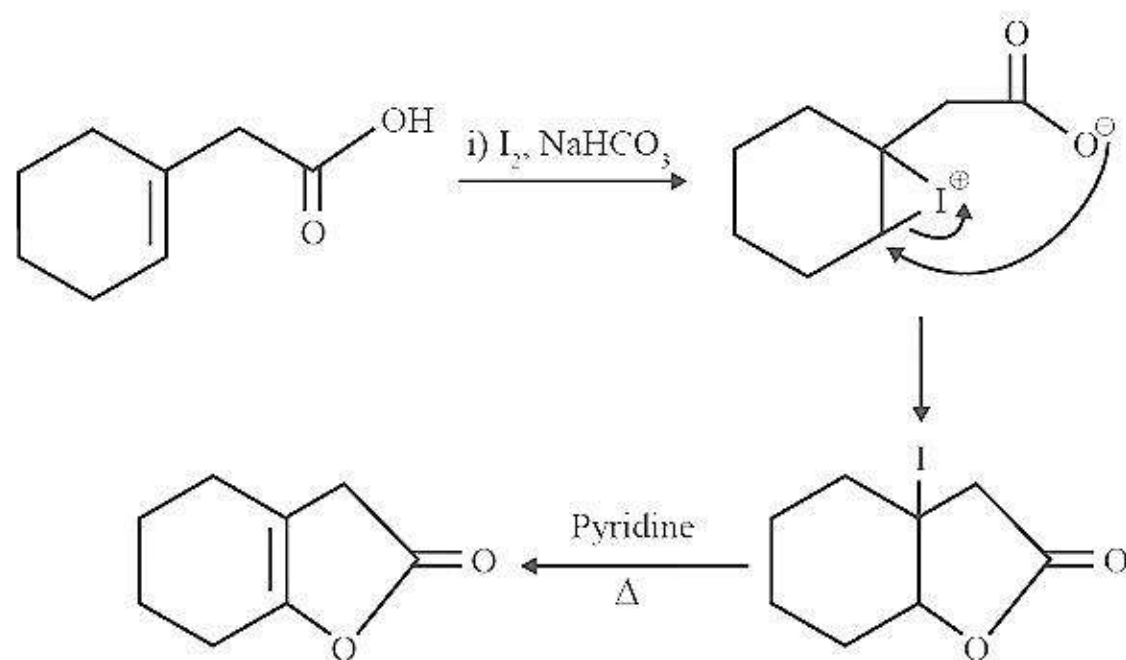


Options:

- (a)
- (b)
- (c)
- (d)

Answer: (a)

Solution:



Question: Match the following.

(Column I) Compounds	(Column II) Hybridizations
(A) Ni(CO)_4	(i) d^2sp^3
(B) $[\text{Ni(CN)}_4]^{2-}$	(ii) sp^3
(C) $[\text{Co(CN)}_6]^{4-}$	(iii) dsp^2

Options:

- (a) $\text{A} \rightarrow \text{(ii)}; \text{B} \rightarrow \text{(iii)}; \text{C} \rightarrow \text{(i)}$
 (b) $\text{A} \rightarrow \text{(iii)}; \text{B} \rightarrow \text{(i)}; \text{C} \rightarrow \text{(ii)}$
 (c) $\text{A} \rightarrow \text{(iii)}; \text{B} \rightarrow \text{(ii)}; \text{C} \rightarrow \text{(i)}$
 (d) $\text{A} \rightarrow \text{(i)}; \text{B} \rightarrow \text{(iii)}; \text{C} \rightarrow \text{(ii)}$

Answer: (a)

Solution:

A) $\text{Ni(CO)}_4 \Rightarrow \text{sp}^3$

B) $[\text{Ni(CN)}_4]^{2-} \Rightarrow \text{dsp}^2$

C) $[\text{Co(CN)}_6]^{4-} \Rightarrow \text{d}^2\text{sp}^3$

Question: Assertion: Aniline on nitration gives o, p, m products

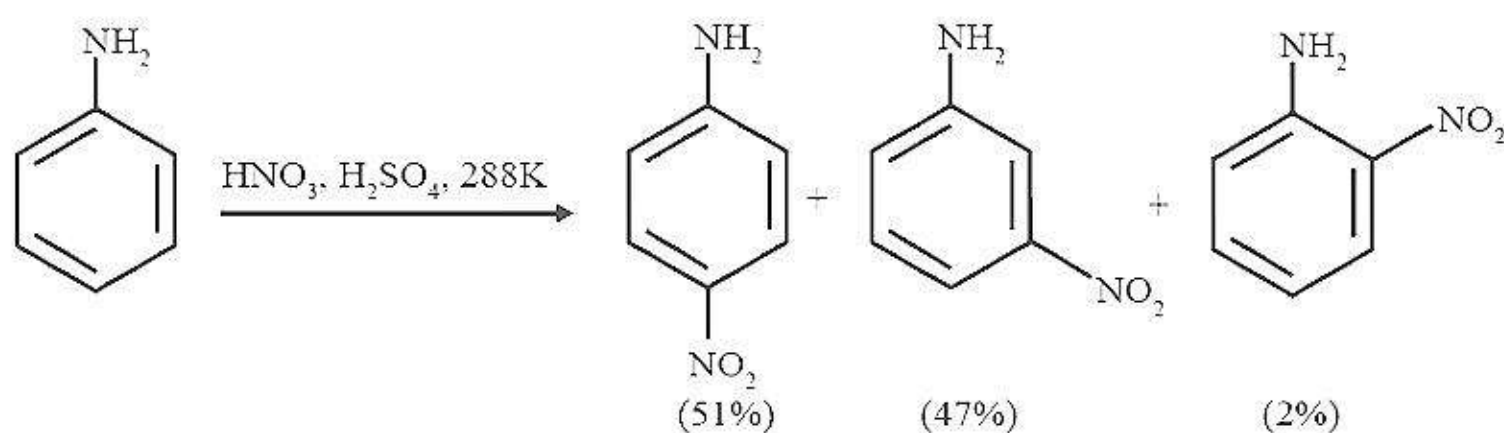
Reason: Mixture used in nitration is acidic

Options:

- (a) Both assertion and reason are true, reason is correct explanation of assertion.
 (b) Both assertion and reason are true, but reason is not a correct explanation of the assertion.
 (c) Assertion is true, but reason is false
 (d) Assertion is false, but reason is true

Answer: (a)

Solution: Direct nitration of aniline yields tarry oxidation products in addition to the nitro derivatives. Moreover, in the strongly acidic medium, aniline is protonated to form the anilinium ion which is meta directing. That is why besides the ortho and para derivatives, significant amount of meta derivative is also formed.



Question: Assertion: Zero orbital overlap is an out of phase overlap

Reason: It results due to different orientation / direction of approach of orbitals

Options:

- (a) Both assertion and reason are true, reason is correct explanation of assertion.
- (b) Both assertion and reason are true, but reason is not a correct explanation of the assertion.
- (c) Assertion is true, but reason is false
- (d) Assertion is false, but reason is true

Answer: (a)

Solution: Zero overlap is out of phase due to different orientation direction of approach

Question: Find the change in oxidation state of oxygen when KIO_4 reacts with H_2O_2 ?

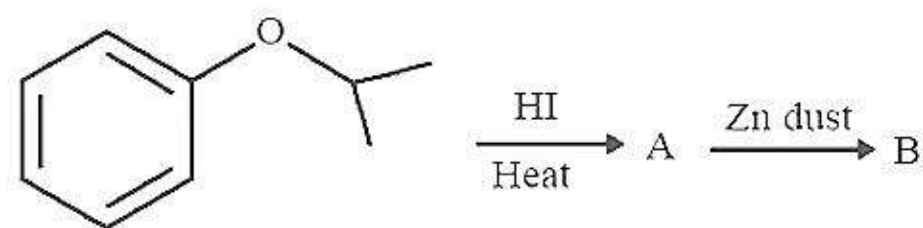
Options:

- (a) 4
- (b) 2
- (c) 1
- (d) 3

Answer: (c)

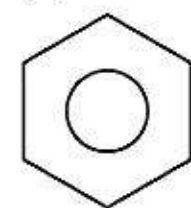
Solution: $\text{KIO}_4 + \text{H}_2\overset{-1}{\text{O}} \rightarrow \text{KIO}_3 + \text{H}_2\text{O} + \overset{0}{\text{O}_2}$

Question:

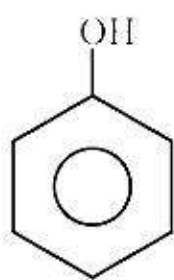


Options:

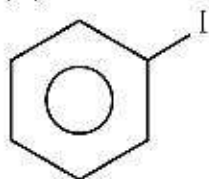
(a)



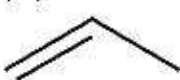
(b)



(c)

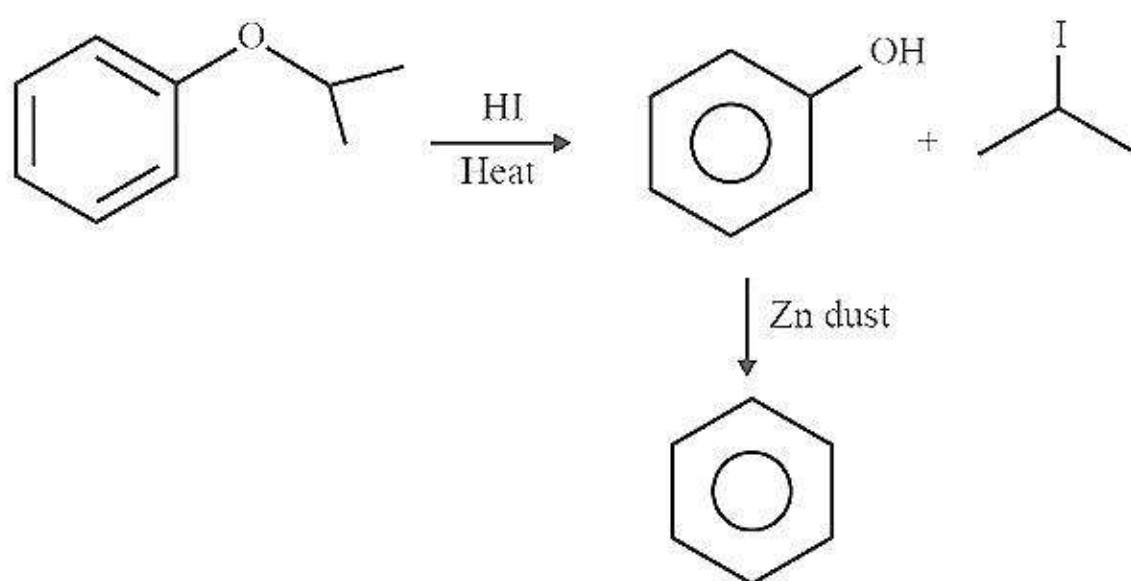


(d)



Answer: (a)

Solution:



Question: Matching of polymers:

(Column I)	(Column II)
(A) Nylon	(i) Thermosetting
(B) Bakelite	(ii) Thermoplastic
(C) Polythene	(iii) Elastomer
(D) Natural rubber	(iv) Fibers

Options:

(a) A $\rightarrow$ (i); B $\rightarrow$ (iii); C $\rightarrow$ (ii); D $\rightarrow$ (iv)

(b) A $\rightarrow$ (iv); B $\rightarrow$ (i); C $\rightarrow$ (ii); D $\rightarrow$ (iii)

(c) A $\rightarrow$ (iii); B $\rightarrow$ (ii); C $\rightarrow$ (iv); D $\rightarrow$ (i)

(d) A $\rightarrow$ (ii); B $\rightarrow$ (i); C $\rightarrow$ (iv); D $\rightarrow$ (iii)

Answer: (b)

Solution:

Nylon $\Rightarrow$ Fibers

Bakelite $\Rightarrow$ Thermosetting

Polythene $\Rightarrow$ Thermoplastic

Natural rubber $\Rightarrow$ Elastomer

Question: Arrange the following according to the rate of nitration:

1) p - xylene

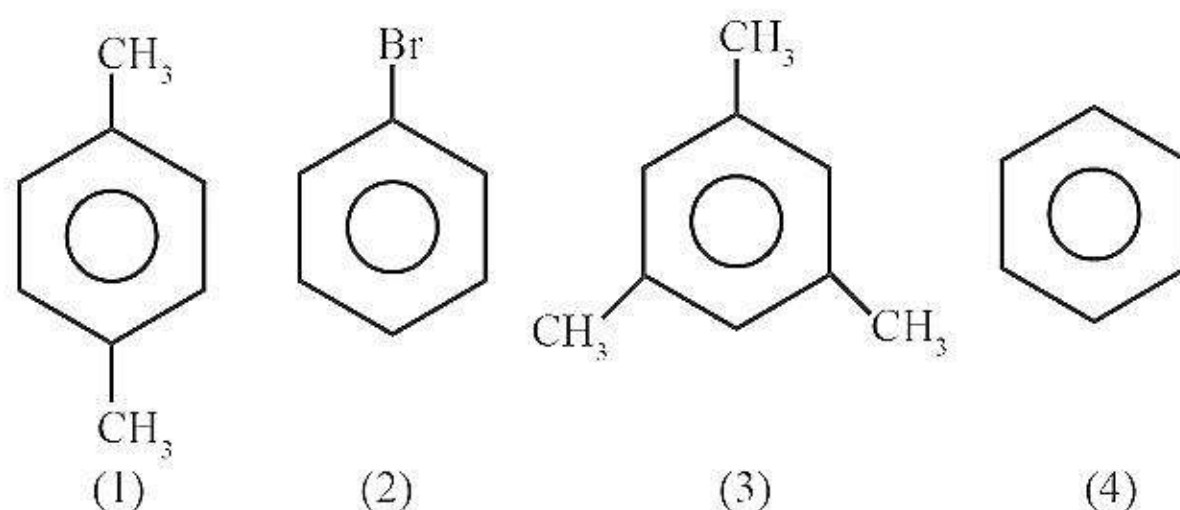
- 2) bromobenzene
 3) mesitylene
 4) benzene

Options:

- (a) $3 > 1 > 4 > 2$
 (b) $2 > 1 > 4 > 3$
 (c) $1 > 3 > 2 > 4$
 (d) $4 > 2 > 3 > 1$

Answer: (a)

Solution:



More is the electron density groups attached, higher is the rate of nitration.

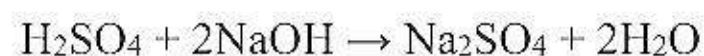
Question: Find the molarity of Na₂SO₄ formed upon mixing of 0.2 M, 2 L H₂SO₄ and 0.1 M, 2 L NaOH.

Options:

- (a) 0.05 M
 (b) 0.03 M
 (c) 0.04 M
 (d) 0.025 M

Answer: (d)

Solution:



Initial moles 0.4 0.2 – –

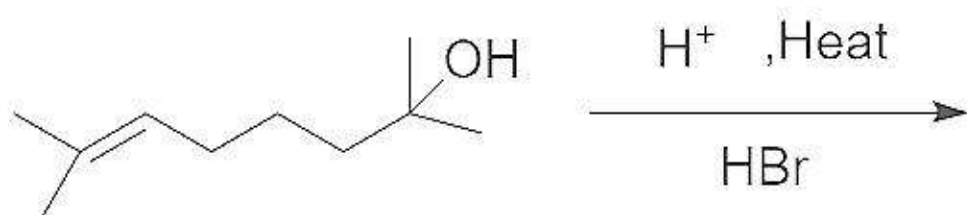
NaOH is the limiting reagent

$\therefore$ 2 moles of NaOH $\rightarrow$ 1 mole of Na₂SO₄

$\therefore$ 0.2 moles of NaOH $\rightarrow \frac{0.2}{2} = 0.1$ moles of Na₂SO₄

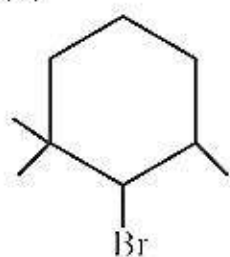
$$M_{\text{Na}_2\text{SO}_4} = \frac{0.1}{4} = 0.025 \text{ M}$$

Question:

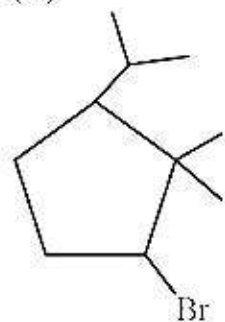


Options:

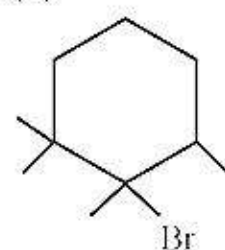
(a)



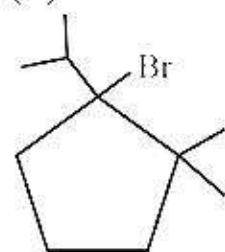
(b)



(c)

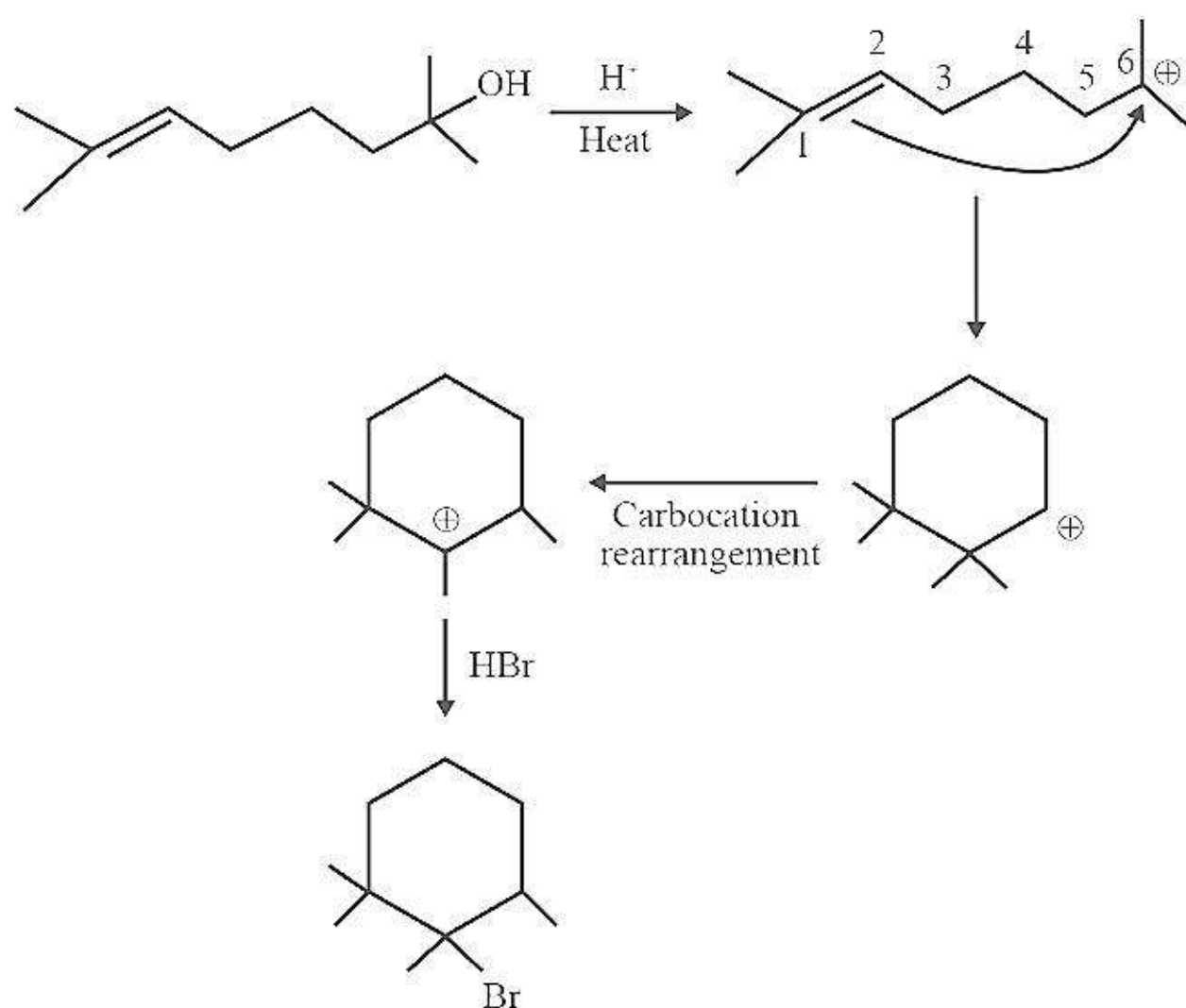


(d)



Answer: (c)

Solution:



Question: The temperature at which rms speed of gas molecules becomes double of its value at 0°C is

Options:

- (a) 819°C
- (b) 760°C
- (c) 273°C
- (d) 224°C

Answer: (a)

Solution:

$$V_{\text{rms}} = \sqrt{\frac{3RT}{M}}$$

$$V_{\text{rms}} \propto \sqrt{T}$$

$$\frac{(V_{\text{rms}})_1}{(V_{\text{rms}})_2} = \sqrt{\frac{T_1}{T_2}}$$

$$(V_{\text{rms}})_2 = 2(V_{\text{rms}})_1$$

$$\frac{1}{2} = \sqrt{\frac{273}{T_2}}$$

$$\Rightarrow T_2 = 273 \times 4 = 1092 \text{ K}$$

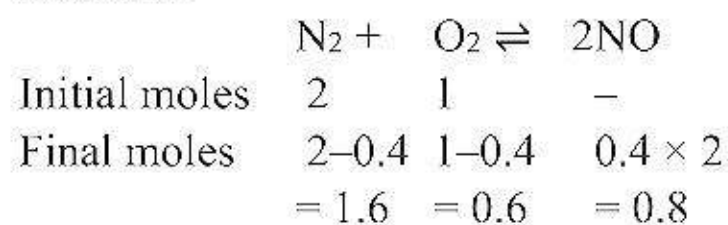
$$= 819^\circ\text{C}$$

Question: In the reaction, $\text{N}_2 + \text{O}_2 \rightleftharpoons 2\text{NO}$

If initial moles of N_2 and O_2 are 2 mol and 1 mol, then find the value of K_c . The moles of O_2 at equilibrium is 0.6 mol.

Answer: 0.67

Solution:



$$K_c = \frac{[\text{NO}]^2}{[\text{N}_2][\text{O}_2]} = \frac{\left(\frac{0.8}{V}\right)^2}{\left(\frac{1.6}{V}\right)\left(\frac{0.6}{V}\right)} = 0.67$$

Question: In a FCC lattice a molecule, density = 9.03 g/cm^3 , $a = 4 \times 10^{-8} \text{ cm}$. Find molecular weight

Answer: 87.00

Solution: $Z = 4$, density = 9.03 cm^{-3} , $a = 4 \times 10^{-8} \text{ cm}$

$$d = \frac{ZM}{a^3 N_a}$$

$$M = \frac{da^3 N_a}{Z} = \frac{9.03 \times (4 \times 10^{-8})^3 \times 6.02 \times 10^{23}}{4} = 869.7 \times 10^{-1}$$

$$= 86.97 \text{ g/mol}$$

$$M \approx 87 \text{ g/mol}$$

Question: How many of the following are not isoelectronic species?

A) Yb^{+3}

B) Tb^{+4}

C) Eu^{+3}

D) Tb^{+2}

Answer: 4.00

Solution:

$\text{Yb}^{+3} - 4f^{13}$ (67 electrons)

$\text{Tb}^{+4} - 4f^7$ (61 electrons)

$\text{Eu}^{+3} - 4f^6$ (60 electrons)

$\text{Tb}^{+2} - 4f^9$ (63 electrons)

None of them is isoelectronic species

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MATHEMATICS

Question: If $P\left(\frac{B}{A}\right) = \frac{5}{7}$; $P\left(\frac{A}{B}\right) = \frac{7}{9}$ and $P(A \cap B) = \frac{1}{9}$. Given

$$S_1 \equiv P(A' \cup B) = \frac{5}{6}, S_2 \equiv P(A' \cap B') = \frac{1}{18}, \text{ then:}$$

Options:

- (a) Both S_1 and S_2 are correct
- (b) S_1 is true and S_2 is false
- (c) S_1 is false and S_2 is true
- (d) Both S_1 and S_2 are false

Answer: (d)

Solution:

$$\text{Given: } \frac{P(A \cap B)}{P(A)} = \frac{5}{7} \text{ \& } \frac{P(A \cap B)}{P(B)} = \frac{7}{9}$$

As $P(A \cap B) = \frac{1}{9}$, we get

$$P(A) = \frac{7}{45} \text{ \& } P(B) = \frac{1}{7}$$

$$P(A' \cap B') = 1 - P(A \cup B)$$

$$= 1 - \left(\frac{7}{45} + \frac{1}{7} - \frac{1}{9} \right)$$

$$= \frac{256}{315}$$

$$\text{Given, } S_1 \equiv P(A' \cup B) = \frac{5}{6}$$

$$S_2 \equiv P(A' \cap B') = \frac{1}{18}, \text{ then:}$$

$$\text{As } P(A \cap B) = \frac{1}{9}, \text{ we get } P(A) = \frac{7}{45} \text{ \& } P(B) = \frac{1}{7}$$

$$P(A' \cup B) = 1 - (P(A) - P(A \cap B))$$

$$= 1 - \left(\frac{7}{45} - \frac{1}{9} \right) = \frac{43}{45}$$

$\therefore S_1$ is wrong.

Question: Absolute maximum value of $f(x) = \tan^{-1}(\sin x - \cos x)$ is:

Options:

(a) 0

(b) $\tan^{-1} \frac{1}{\sqrt{2}} - \frac{\pi}{4}$

(c) $\frac{\pi}{4}$

(d) $\tan^{-1} \sqrt{2}$

Answer: (d)

Solution:

$$\therefore \sin x - \cos x \in [-\sqrt{2}, \sqrt{2}]$$

So maximum value of $\tan^{-1}(\sin x - \cos x)$ is $\tan^{-1} \sqrt{2}$

Question: The values of λ for which the lines $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z+3}{\lambda^2}$ and $\frac{x-3}{1} = \frac{y-2}{\lambda^2} = \frac{z-1}{2}$ are coplanar, are:

Options:

(a) $\pm\sqrt{3}$

(b) $\pm\sqrt{5}$

(c) ± 2

(d) $\pm\sqrt{2}$

Answer: (d)

Solution:

For coplanarity

$$\begin{vmatrix} 1 & 2 & \lambda^2 \\ 1 & \lambda^2 & 2 \\ 2 & 0 & 4 \end{vmatrix} = 0$$

$$\Rightarrow 1(4\lambda^2) - 2(0) + \lambda^2(-2\lambda^2) = 0$$

$$\Rightarrow -2\lambda^4 + 4\lambda^2 = 0$$

$$\Rightarrow \lambda^2(\lambda^2 - 2) = 0$$

$$\Rightarrow \lambda = \pm\sqrt{2}$$

Question: From point $(2, 0)$ tangents are drawn on $2y^2 = -x$. These tangents also touches the circle $(x-5)^2 + y^2 = r^2$. The value of $17r^2$ is:

Options:

(a) 1

(b) 12

(c) 9

(d) 4

Answer: (c)

Solution:

$$P \equiv y^2 = \frac{-x}{2}$$

$$\text{Equation of tangent} \equiv y = mx + \left(-\frac{1}{8m}\right)$$

$$16m^2 = 1 \Rightarrow m = \pm \frac{1}{4}$$

$$\therefore \text{ Tangent are } y = \frac{1}{4}x - \frac{1}{2} \text{ \& } y = -\frac{x}{4} + \frac{1}{2}$$

Equation of tangent to $(x-5)^2 + y^2 = r^2$ are:

$$x - 4y = 2 \text{ \& } 4y + x = 2$$

Using $d_c = r$, we get

$$\left| \frac{5-0-2}{\sqrt{17}} \right| = r \text{ or } \left| \frac{5+0-2}{\sqrt{17}} \right| = r$$

$$\therefore r^2 = \frac{9}{17}$$

$$\Rightarrow 17r^2 = 9$$

Question: Let $f(x) = \lim_{n \rightarrow \infty} \frac{\cos 2\pi x - x^{2n} \sin(x-1)}{1 + x^{2n+1} - x^{2n}}$, is continuous at:

Options:

(a) $R - \{1\}$

(b) $R - \{-1, 1\}$

(c) $R - \{0, 1\}$

(d) $R - \{0\}$

Answer: (b)

Solution:

$$\text{For } |x| < 1, f(x) = \cos 2\pi x$$

$$\text{For } |x| > 1, f(x) = -\frac{\sin(x-1)}{x-1}$$

$$\text{For } |x| = 1, f(x) = \begin{cases} 1 & \text{if } x = 1 \\ \frac{1 + \sin 2}{-1} & \text{if } x = -1 \end{cases}$$

$$\lim_{x \rightarrow 1^+} f(x) = -1, \lim_{x \rightarrow 1^-} f(x) = 1$$

So f is discontinuous at $x = 1$

$$\lim_{x \rightarrow -1^+} f(x) = 1, \lim_{x \rightarrow -1^-} f(x) = -\frac{\sin 2}{2}$$

So $f(x)$ is discontinuous at $x = -1$

Question: A class have B boys and G girls, 3 boys and 2 girls selected at random and number of ways of selecting 3 boys and 2 girls are 168. Then $B + 3G$ is equal to _____.

Answer: 17.00

Solution:

Given that

$${}^B C_3 \cdot {}^G C_2 = 168$$

$$\Rightarrow \frac{B(B-1)(B-2)}{6} \cdot \frac{G(G-1)}{2} = 168$$

$$\Rightarrow B(B-1)(B-2)G(G-1) = 7 \cdot 6 \cdot 4 \cdot 3 \cdot 2 \cdot 2$$

$$\Rightarrow B(B-1)(B-2)G(G-1) = 8 \cdot 7 \cdot 6 \cdot 3 \cdot 2$$

$$\therefore B = 8 \text{ \& } G = 3$$

$$\Rightarrow B + 3G = 8 + 9 = 17$$

Question: Let $f(x) = ax^2 + bx + c$ and $f(1) = 3$, $f(-2) = \lambda$, $f(3) = 4$, then the value of λ for which $f(0) + f(1) + f(-2) + f(3) = 14$ is _____.

Answer: 4.00

Solution:

$$\text{Given, } f(x) = ax^2 + bx + c$$

$$f(0) = c$$

$$f(1) = a + b + c = 3 \quad \dots(1)$$

$$f(-2) = 4a - 2b + c = \lambda \quad \dots(2)$$

$$f(3) = 9a + 3b + c = 4 \quad \dots(3)$$

By solving (2) & (3)

$$a + b = \frac{4 - \lambda}{5}$$

$$\text{Also, } c = 3 + \lambda + 4 = 14$$

$$c = 7 - \lambda$$

$\therefore$ Putting in (1)

$$\frac{4 - \lambda}{5} + 7 - \lambda = 3$$

$$6\lambda = 24$$

$$\lambda = 4$$